PERGAMON

Journal of Structural Geology 21 (1999) 719–727

JOURNAL OF STRUCTURAL GEOLOGY

Fold symmetry—a quantitative description

Amit Tripathi, V.K. Gairola*

Department of Geology, Banaras Hindu University, Varanasi 221 005, India

Received 10 December 1998; accepted 9 March 1999

Abstract

The study of symmetry of a fold has so far been limited to a qualitative distinction between symmetrical or asymmetrical folds. There exists no satisfactory scheme of quantitative definition of the degree of symmetry or asymmetry of a fold. The present paper attempts to address this problem by proposing a classification of folds based on a concept of '*degree of asymmetry*' deduced from Fourier coefficients defining the shape and size of the two limbs of the fold.

1. Introduction

Several definitions of symmetric folds have been given in the geological literature. The most comprehensive one however, has been given by Ramsay (1967, p. 357). According to this "*complete definition* of symmetric folds…*the axial surface must be the right bisector of a line drawn between two adjacent inflexion points and this bisector must divide the fold into two identical (and mirror image) parts*." This definition groups all folds into two categories, viz. symmetric folds and asymmetric folds. In nature however, perfect symmetric folds are rare and thus a quantitative classification of folds symmetry would have a greater practical application than just calling them symmetric or asymmetric. Therefore, in this paper we propose a method of quantifying the asymmetry of a folded surface between two consecutive inflexion points and measure it in the form of *degree of asymmetry* using Fourier analysis of fold profiles.

The use of Fourier series for the study of folds is not new. It was suggested by Norris (1963) and different methods and applications have been subsequently described, for example, by Harbaugh and Preston (1965), Whitten (1966), Chapple (1968), Stabler (1968), Hudleston (1973), Singh and Gairola (1992) and Srivastava and Gairola (1997).

A Fourier series is an infinite series of trigonometric terms. Because of the periodicity of trigonometric functions, this series is useful in the investigation of several periodic physical phenomena including folds. The Fourier series can be represented by:

$$\frac{a_0}{2} + \sum_{n=1}^{\infty}[a_n \cos(nx) + b_n \sin(nx)] \tag{1}$$

where a_n and b_n are constants, n is an integer and x is a variable with value between 0 and π.

For a quarter wavelength unit of a fold, all cosine terms, even number terms and the constant (a_0) are zero (Stabler, 1968) and the effective Fourier series thus becomes:

$$\sum_{n=1}^{\infty} b_n \sin(nx) \tag{2}$$

where n is an odd integer. This series represents a general curve which will describe any fold shape between a point of zero curvature (inflexion point) and the point of maximum curvature (hinge point).

The evaluation of the coefficients (b_n) of the Fourier series has been dealt with by several workers. However, we propose the use of the method proposed by Stabler (1968). Singh and Gairola (1992) have

* Corresponding author.
E-mail address: gairola@banaras.ernet.in (V.K. Gairola)

0191-8141/99/$ - see front matter
PII: S0191-8141(99)00066-8

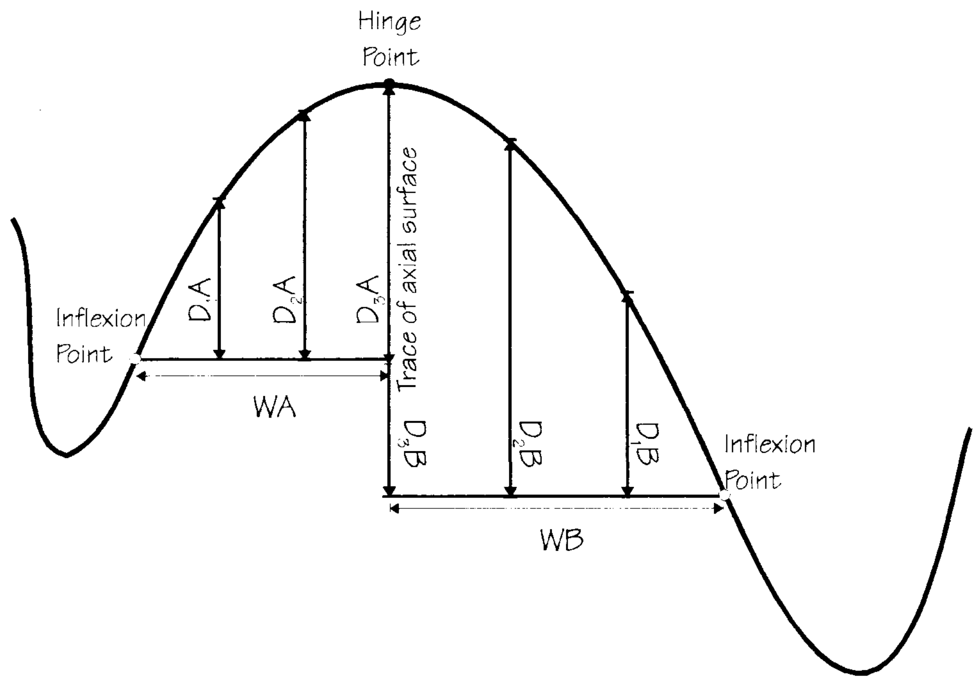


Fig. 1. Measurement of data for Fourier analysis.

suggested that for elimination of scale from measurements proposed by Stabler (1968), the measured D_n values must be corrected by multiplying the measured lengths by a factor of $\pi/2W$; W being the length of quarter wave base.

According to this method a line from the inflexion point is drawn normal to the trace of the axial surface (Fig. 1) and its length (W) measured. The distance of the fold profile from three equally spaced points (Fig. 1) is also measured (D_1, D_2 and D_3). The scale-corrected values of these distances (Y_1, Y_2 and Y_3) can now be obtained by the expression:

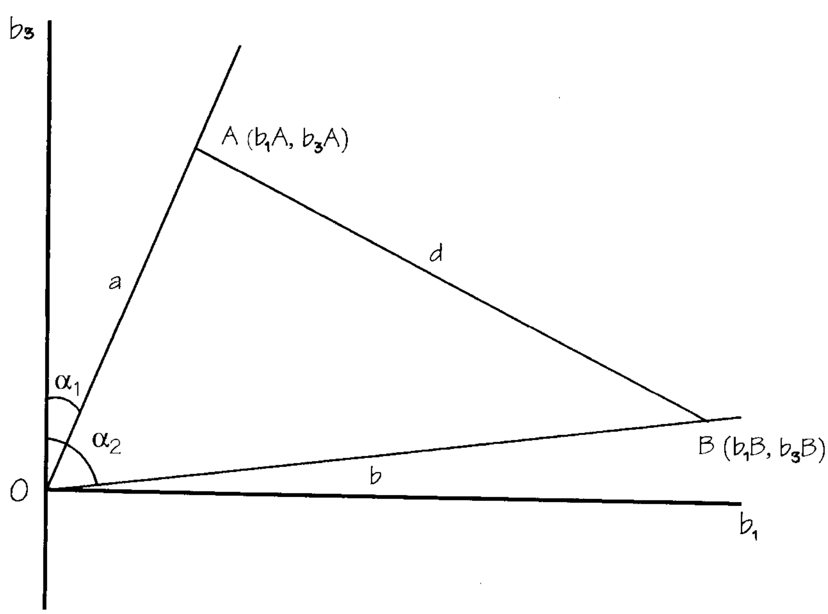


Fig. 2. Measurement of the degree of asymmetry. Points A and B represent the two limbs of a fold on the b_1 vs b_3 graph. The angle $|\alpha_1 - \alpha_2|$ represents the shape difference between the two limbs.

$$Y_n = D_n \frac{\pi}{2W} \tag{3}$$

where $n = 1,2,3$. Stabler (1968) has suggested a very simple expression for evaluation of the first two odd sine coefficients of the Fourier series as:

$$b_1 = \frac{Y_3 + Y_1}{3} + \frac{Y_2}{2 \sin 60^\circ} \tag{4}$$

$$b_3 = \frac{2Y_1 + Y_3}{3} + \frac{1}{2 \sin 60^\circ}. \tag{5}$$

As shown above, Fourier coefficients provide a method for quantifying the shape of a folded surface. Hudleston (1973) has used this principle to classify folds into six categories of ideal styles (A–F) based on their b_3/b_1 ratio. He has also suggested that asymmetry of folds may be evaluated by comparison of coefficients of the two halves of a fold. In the present paper, the information from the b_3 vs b_1 graph of Hudleston (1973) has been used to define quantitatively the asymmetry of a fold.

2. Principle

The shape of quarter wave sector of a folded surface can be represented by a point on the b_1 vs b_3 graph. Thus the left and right limbs of a single folded surface can be represented by two specific points (Hudleston, 1973, fig. 9). The relative position of these two points on the b_3/b_1 graph will represent the difference in geometry of the two surfaces (Fig. 2). If the two points have identical b_1 and b_3 co-ordinates the limbs would have identical shape and size and the fold they represent is a symmetrical fold. However, if the two points are separate but lie along the same line joining them to the origin, the two limbs will have the same shape but different size (amplitude). The degree of asymmetry of a folded surface, from one inflexion point to another, is thus a function of the distance between the two points (d) representing the two limbs on the b_1 vs b_3 graph (A and B) from the origin and the angle between the lines $|\alpha_2 - \alpha_1|$ joining these points to the origin (Fig. 2).

Fig. 2 shows two points A and B with b_1 and b_3 co-ordinates as (b_1A, b_3A) and (b_1B, b_3B), respectively. These two points form a triangle OAB with the origin. Lengths of the lines OA (=a) and OB (=b) can be given by:

$$a = \sqrt{(b_1A)^2 + (b_3A)^2} \tag{6}$$

$$b = \sqrt{(b_1B)^2 + (b_3B)^2}. \tag{7}$$

Table 1
Fold classification based on profile symmetry

$D_A = 0$	Symmetrical
$0 < D_A \leq 3$	Sub-symmetrical
$3 < D_A \leq 6$	Slightly asymmetrical
$6 < D_A \leq 9$	Moderately asymmetrical
$9 < D_A \leq 12$	Highly asymmetrical
$12 < D_A$	Extremely asymmetrical

The distance (d) between the points A and B is given by:

$$d = \sqrt{(b_1A - b_1B)^2 + (b_3A - b_3B)^2}. \tag{8}$$

The difference in size (amplitude) of the two limbs is represented by their distance from the origin. A standardised measure of the difference in the amplitude of the two limbs can thus be defined as:

$$\Delta_{\text{size}} = \left| \frac{d}{a} - \frac{d}{b} \right|. \tag{9}$$

The angles that the lines OA and OB make with the b_3 axis (α_1 and α_2, respectively in Fig. 2) can be given as:

$$\alpha_1 = \tan^{-1}\left(\frac{b_1A}{b_3A}\right) \tag{10}$$

$$\alpha_2 = \tan^{-1}\left(\frac{b_1B}{b_3B}\right). \tag{11}$$

The difference in shape of the two limbs is represented as the angle between the lines joining them to the origin. The shape component (Δ_{shape}) can thus be defined as:

$$\Delta_{\text{shape}} = |\alpha_1 - \alpha_2|. \tag{12}$$

As defined earlier, the degree of asymmetry (D_A) of a folded surface is a function of two variables. It is therefore being defined as the sum of these components, that is, the shape component (Δ_{shape}) and the size component (Δ_{size}):

$$D_A = \Delta_{\text{shape}} + \Delta_{\text{size}}. \tag{13}$$

Based on the above derivation, we propose to define asymmetric folds as folds where the shapes of the quarter wave sectors (from hinge to inflexion point) on either side of the fold are not mirror images of each other and/or the amplitude of the quarter wave on both the limbs of the folded surface are not equal. We propose to measure this asymmetry as the sum of the shape difference between the limbs (Δ_{shape}) and the standardised amplitude difference between the limbs (Δ_{size}) as represented by the Fourier coefficients. That

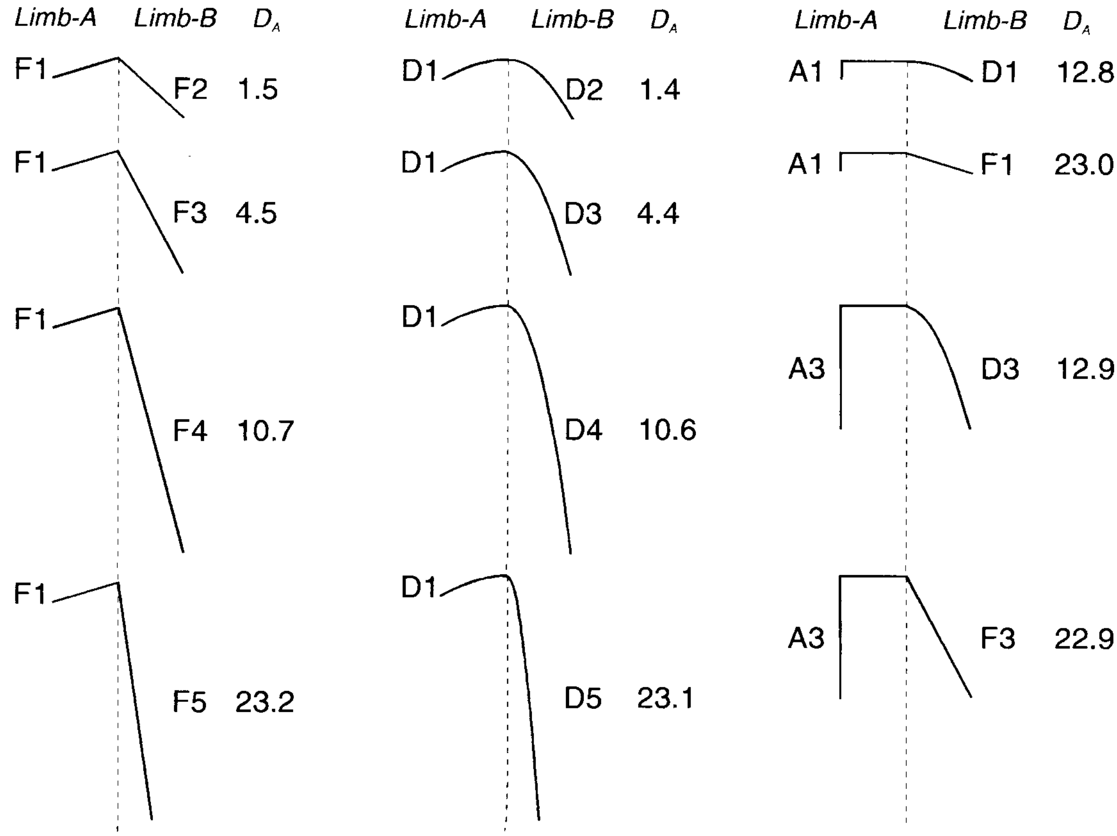


Fig. 3. Asymmetry variation in hypothetical folds resulting from combination of ideal fold forms of Hudleston (1973). First two columns represent folds generated by combining folds of identical shape category but belonging to different categories of amplitude. The third column contains folds generated from combination of folds having the same amplitude but belonging to different categories of shape.

is, the differences in shape and size of the limbs affect the asymmetry equally. We propose to define asymmetry of a folded surface as *the difference in size and shape of two limbs of a fold between two consecutive inflexion points*.

Based on the degree of asymmetry, folds can be classified into groups having a comparable range of asymmetry. We propose to classify folds into the following six categories (Table 1).

3. Example

To assess the application of this method, it was used to determine the asymmetry of different categories of folded surfaces. The method was applied to ideal fold shapes, experimentally developed parallel folds as well as natural similar folds. The latter two classes have been chosen because they represent two fundamentally different fold geometries.

3.1. Ideal fold forms

Based on his visual harmonic analysis, Hudleston (1973) has described 30 *ideal fold forms* characterised by their shape and amplitude. He described six categories of shape (A, B, C, D, E and F) and five categories of amplitude (1, 2, 3, 4 and 5). We have calculated the degree of asymmetry of hypothetical folds that would result by combining these *ideal fold forms* (Fig. 3). Hypothetical folds were constructed by combining folds of shape categories F (chevron), D (parabola) and A (box) and amplitude categories 1, 2, 3, 4 and 5. Folds having the same shape but different amplitudes were combined to separate out the effect of amplitude (size) on asymmetry while folds having identical amplitude but different shape were combined to study the effect of shape on the asymmetry. The folds resulting from these combinations are shown in Fig. 3 along with their D_A values. It is clear from this figure that the asymmetry due to difference in limb size

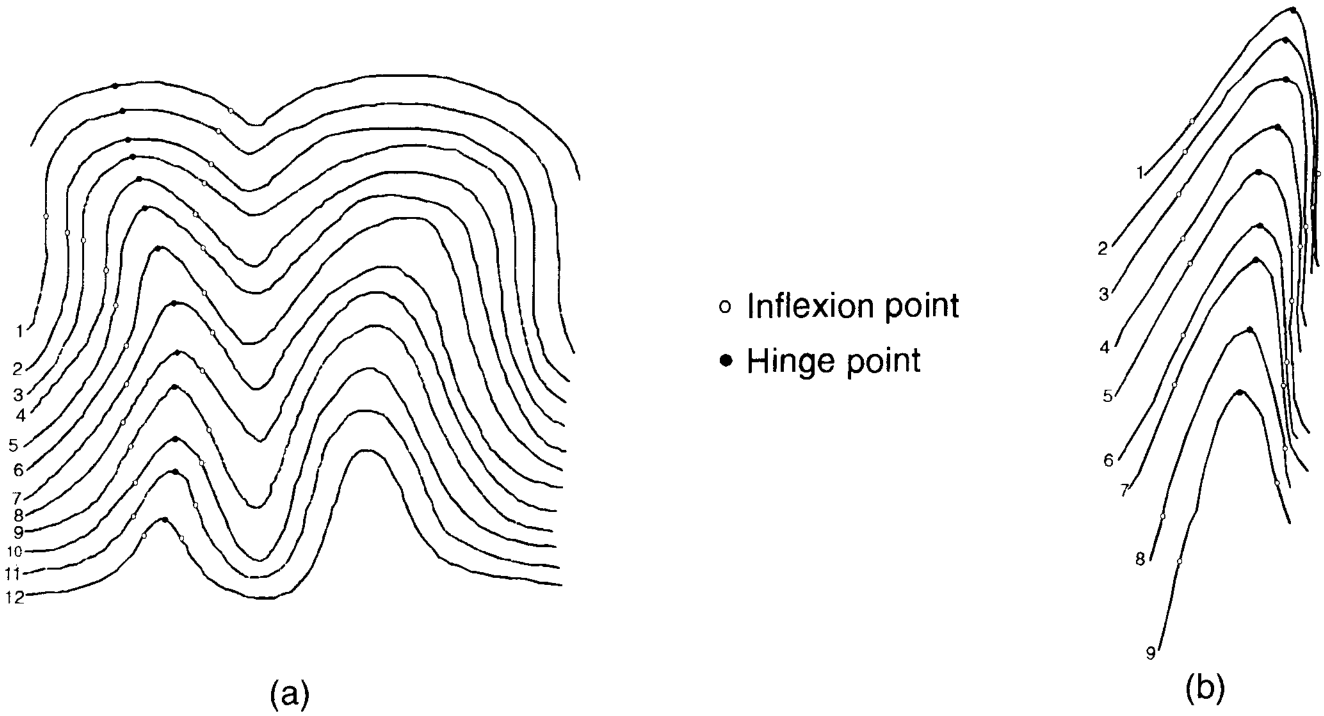
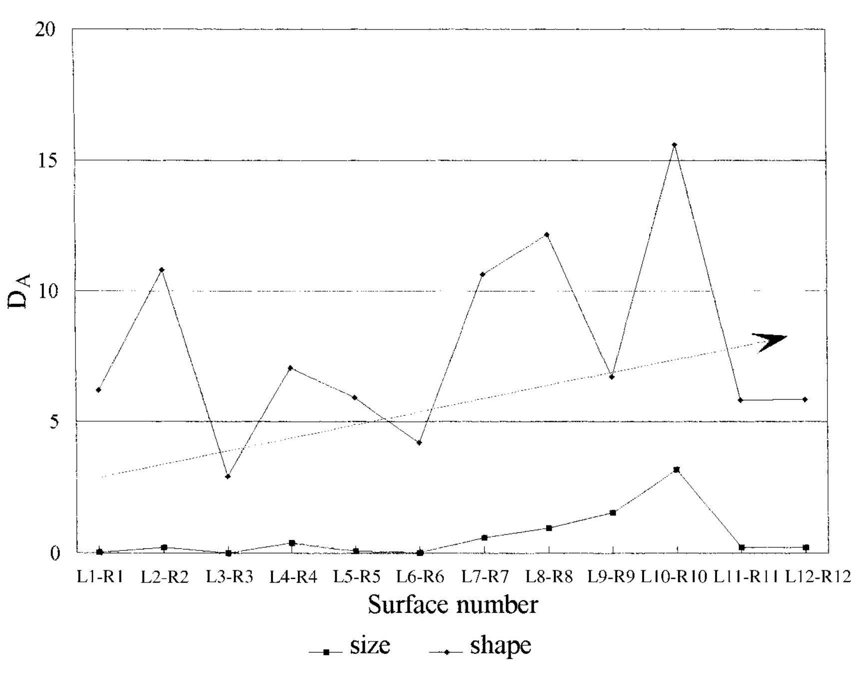


Fig. 4. (a) Experimentally developed fold shown in fig. 4(c) of Srivastava and Gairola (1988). The outermost surface of the almost parallel fold has not been selected because the left inflexion point can not be reliably marked. Below surface 12 of the fold, a set of layers oriented oblique to the folded layers occur. The interference of boundary conditions between these two differently oriented layers may be the reason for the different asymmtetry conditions in surfaces 11 and 12. (b) A natural similar fold in quartzite used here for asymmetry estimation.

(resulting from combination of extreme size classes F1–F5 and D1–D5) is similar to the asymmetry due to difference in limb shape (resulting from combination of extreme shape classes A1–F1 and A3–F3).

3.2. Experimentally developed parallel folds

The method was applied to experimentally developed folds in plasticine by Srivastava and Gairola



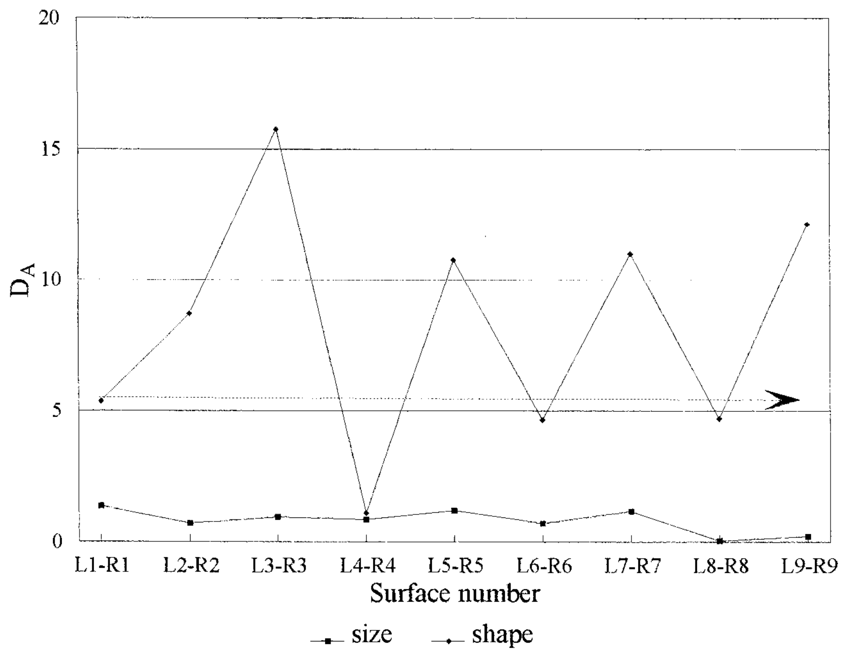


Fig. 6. D_A variation across a natural similar fold shown in Fig. 4(b). Arrow denotes the general trend of D_A.

(1988). The fold in fig. 4(c) of their work has been selected for the present study (Fig. 4a). It was observed that the asymmetry shows a slight increase from outer to the inner arcs of the fold (Fig. 5) with the exception of the innermost layer that shows lower asymmetry, perhaps due to interference of boundary conditions with adjoining oblique layers. The asymmetry also shows a regular pattern of increase and decrease. The Δ_{size} component generally increases toward the inner arc.

3.3. Naturally occurring similar folds

Asymmetry determination was carried out on natural similar folds (Fig. 4b) developed in quartzite. In contrast to the analysis of parallel folds, it was observed that the asymmetry of layers does not follow an increasing trend towards inner arc (Fig. 6). The D_A values however do show a cyclic pattern of increase and decrease especially in the Δ_{shape} component with the Δ_{size} component varying only within narrow limits (Fig. 6).

4. Discussion

The D_A value has been defined as a sum of two variables (Δ_{shape} and Δ_{size}). This scheme gives equal weight to changes in size and shape in determining the asymmetry. In other words the degree of asymmetry of a fold is equally sensitive to changes in shape and size of the limbs. However, significantly different combinations of shape and size may produce the same asymmetry.

In Fig. 5 the degree of asymmetry progressively increases towards the inner arc of the fold. This may perhaps be attributed to the fold having a similar shape and a changing amplitude across the layers which is a property of parallel folds. However from the results we may add that this change in amplitude too is not constant but increases towards inner arc. In similar folds the change in amplitude is almost constant throughout the fold (Fig. 6) but the shape changes sharply across the fold—a property of similar folds.

Twiss (1988) has studied the geometrical properties of folds and has given a detailed classification of perfect symmetric folds using the style elements of bluntness, folding angle and aspect ratio—in half wavelength of a fold, i.e. between two consecutive inflexion points. This method can be extended to certain categories of imperfect symmetric folds and asymmetric folds and is very useful in understanding the geometry of folds and style of folding. However, application of this model to asymmetric folds requires additional geometrical construction and evaluation of more parameters "*...asymmetric folds thus require twice the number of parameters to define their geometry as symmetric folds, and because of this additional complexity less attention is generally given to the study of the geometry of asymmetric folds*" (Twiss, 1988, p. 620). The model being proposed in the present paper has addressed this particular problem of studying the asymmetric folds. The present model treats a quarter

Table 2
α values corresponding to Hudleston's (1973) fold classes

α_1 or α_2 (in degrees)	Fold style
71.6	Box fold
80.6	Semi-ellipse fold
87.9	Parabola fold
90.0	Sine wave fold
96.3	Chevron fold

wave sector of a fold—from inflexion point to hinge—as one fold unit (as opposed to half wave sector of Twiss, 1988) and mathematically compares the parameters of shape and size from two distinct fold units. Since comparison is completely mathematical the model is not prone to subjectivity. However, since the method is based on the Fourier technique, all its provisions apply to the present model as well.

The quantitative measure of the asymmetry of a fold is a new technique and may find application in several geological studies. Some of the possible applications are mentioned below:

- The D_A value provides a useful method of mathematically comparing the geometry of two folded surfaces. The concept may be of use in any study involving comparison of folds.
- Ramsay (1958, 1962, 1967) and Turner and Weiss (1963) have used the symbol 'M' for symmetric fold and 'S' and 'Z' to represent asymmetric folds on regional geological maps. It has been argued that the fold symmetry in a mesoscopic fold would vary as a function of the distance of the fold from the regional fold hinge zone. We propose to further refine this method by incorporating D_A values on such regional maps and presenting the asymmetry by symbols like S2.4 or Z3.2 which would indicate the vergence as well as amount of asymmetry. Such maps may be of help in structural studies of the area and may help in accurate demarcation of elements of regional folds.
- Quantitative representation of fold style may also be mapped along with the D_A which is partly a measure of style difference between the two limbs of a fold. The values of α_1 and α_2 are in fact the measure of the fold style. Using the fold classification given in table 1 of Hudleston (1973) we can give the fold style as a function of α_1 or α_2 (Table 2).

However, we do recommend the more detailed classification of fold styles derived from the fold classification scheme given by Singh and Gairola (1992). This scheme is also derived from Hudleston's (1973) classification and the α values corresponding to different fold styles has been calculated. For $\alpha<67.8^\circ$ the fold will be a multiple fold and for $\alpha > 99.8^\circ$ the fold will be a cuspate fold. All other categories will have α within this range (i.e. 67.8–99.8°).

Acknowledgements

We are thankful to Dr R.J. Lisle, Dr R.J. Twiss and Dr F. Bastida for their review, constructive comments and suggestions in improving the manuscript. One of the authors (AT) gratefully acknowledges financial support from the Council of Scientific and Industrial Research, Government of India, in the form of a Research Associateship. We also thank the Geological Survey of India Training Institute, Hyderabad, India, for providing library facilities.

Appendix

For a rapid calculation of the degree of asymmetry, from b_1, b_3 and W data for both the fold limbs, a computer program has been developed. The program has been written in MS-QBASIC using MS-DOS 6.22 and a 80486 based PC (to obtain a copy of the program send a blank 3.5″ floppy disk). Data for the program has to be supplied in an ASCII text file having the data format:

```
"FOURIER"
Name of surface a,D1A,D2A,D3A,WA,Name of surface b,D1B,D2B,D3B,WB
...
...
[Data items separated by comma without leading or trailing spaces]
```

The surface names may be alphanumeric while all the other variables are double precision numeric values. The program prompts for output devices from the user and directs the output to the requested device (console, printer or file). The printer device defaults to *lpt1*. The file device prompts for a file name and appends the output at the end of the requested file in ASCII text format without affecting any existing data in the file.

```
'SYMMETRY.BAS - Program from determination of the fold symmetry from Fourier
' data of D1, D2, D3 and W of successive data. The Fourier coefficients b3 and
' b1 are calculated from method of Stabler (1968) and described by Singh and
' Gairola (1992).
DEFDBL A-Z
CLS
PRINT "Enter Output Device "
PRINT "(Console, Printer, File)"
DO
  DO
    outdev$ = INKEY$
  LOOP UNTIL LEN(outdev$)
  outdev$ = UCASE$(outdev$)
LOOP UNTIL outdev$ = "F" OR outdev$ = "C" OR outdev$ = "P"
SELECT CASE outdev$
  CASE "C"
    dev$ = "con"
  CASE "P"
    dev$ = "lpt1"
  CASE "F"
    INPUT "File Name "; filsp$
    dev$ = filsp$
CASE ELSE
END SELECT
OPEN dev$ FOR APPEND AS #2
10
l = 0
INPUT "Fourier Data File Name   "; fil$
OPEN fil$ FOR INPUT AS #1
INPUT #1, d$
IF NOT d$ = "FOURIER" THEN
  BEEP
  PRINT "Incorrect File Format"
  GOTO 10
END IF

PRINT #2, "--------------------------------------------------------------------"
PRINT #2, "surface no.   a      b      d       α1      α2      Δsize   Δshape   DA"
PRINT #2, "--------------------------------------------------------------------"
PRINT #2, UCASE$(fil$)
DO WHILE NOT EOF(1)
  INPUT #1, snoa$, d1, d2, d3, wa
  YA1 = (d1 * 3.142857) / (2 * wa)
  YA2 = (d2 * 3.142857) / (2 * wa)
  YA3 = (d3 * 3.142857) / (2 * wa)
  b1A = (YA1 + (1.73205 * YA2) + YA3) / 3
  b3A = ((2 * YA1) - YA3) / 3
  INPUT #1, snob$: IF EOF(1) THEN EXIT DO
  INPUT #1, d1, d2, d3, wb
  YA1 = (d1 * 3.142857) / (2 * wb)
  YA2 = (d2 * 3.142857) / (2 * wb)
  YA3 = (d3 * 3.142857) / (2 * wb)
  b1B = (YA1 + (1.73205 * YA2) + YA3) / 3
  b3B = ((2 * YA1) - YA3) / 3
  a = SQR((b1A * b1A) + (b3A * b3A))
  b = SQR((b1B * b1B) + (b3B * b3B))
  d1 = (b1A - b1B) ^ 2
  d2 = (b3A - b3B) ^ 2
  d = SQR(d1 + d2)
```

```
    Dsize = ABS((d / a) - (d / b))
    IF SGN(b3A) = 1 THEN
      alpha1 = 57.29578 * (ATN(b1A / ABS(b3A)))
    ELSEIF SGN(b3A) = 0 THEN
      alpha1 = 90
    ELSE
      alpha1 = 90 + (57.29578 * ATN((ABS(b3A)) / b1A))
    END IF
    IF SGN(b3B) = 1 THEN
      alpha2 = 57.29578 * (ATN(b1B / ABS(b3B)))
    ELSEIF SGN(b3B) = 0 THEN
      alpha2 = 90
    ELSE
      alpha2 = 90 + (57.29578 * ATN((ABS(b3B)) / b1B))
    END IF
    Dshape = ABS(alpha1 - alpha2)
    DA = Dshape + Dsize
    PRINT #2, snoa$; " - "; snob$; "  is ";
    PRINT #2, USING "####.##"; a; b; d; alpha1; alpha2; Dsize; Dshape; DA
  LOOP
  CLOSE
  END
```

References

Chapple, W.M., 1968. A mathematical theory of finite amplitude folding. Geological Society of America Bulletin 79, 47–68.

Harbaugh, J.W., Preston, F.W., 1965. Fourier series analysis in geology. 6th Symposium on Computer Applications in Mineral Exploration, Tuscon, Ariz., R1–R46.

Hudleston, P.J., 1973. Fold morphology and some geometrical implications of theories of fold development. Tectonophysics 16, 1–46.

Norris, D.K., 1963. Shearing strain in simple folds in layered media. Geological Society of Canada Paper 63, 26–27.

Ramsay, J.G., 1958. Superimposed folding at Loch Monar, Inverness-shire and Ross-shire. Quarterly Journal of the Geological Society 113, 271–307.

Ramsay, J.G., 1962. Interference pattern produced by superposition of folds of 'Similar' type. Journal of Geology 60, 466–481.

Ramsay, J.G., 1967. Folding and Fracturing of Rocks. McGraw-Hill, New York.

Singh, R.A., Gairola, V.K., 1992. Fold shape analysis on the vicinity of North Almora Thrust in District Chamoli, Garhwal Himalaya. Journal of Himalayan Geology 3, 121–129.

Srivastava, H.B., Gairola, V.K., 1988. Experimental deformation of two sets of obliquely oriented multilayers. Bulletin of the Indian Geologists Association 21, 59–69.

Srivastava, V., Gairola, V.K., 1997. Classification of multilayered folds based on harmonic analysis: Example from central India. Journal of Structural Geology 19, 107–112.

Stabler, C.L., 1968. Simplified Fourier analysis of fold shapes. Tectonophysics 6, 343–350.

Turner, F.J., Weiss, L.E., 1963. Structural Analysis of Metamorphic Tectonites. McGraw-Hill, New York, 545 pp.

Twiss, R.J., 1988. Description and classification of folds in single surfaces. Journal of Structural Geology 10, 607–623.

Whitten, E.H.T., 1966. Structural Geology of Folded Rocks, 3rd ed. Rand McNally, Chicago.